# KINEMATICS OF THE CURRENT LAYER IN A PLASMA ACCELERATOR

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The present paper contains a comparison of experimental and theoretical dependences of the kinematic characteristics of quasi-neutral charged-particle bunches produced in track-type plasma accelerators on the electrical and geometric parameters of the accelerating circuit.

## §1. MOTION OF THE CURRENT LAYER ALONG ERODABLE ELECTRODES

In [1] the state of a plasma bunch of variable mass was described by means of the kinetic potential taking account of the kinetic energy of the moving plasma $W_v$, the energy of the intrinsic magnetic field $W_L$, the Lorentz force function $W_B$, and the electrical energy $W_C$ stored in the accelerating circuit capacitor,

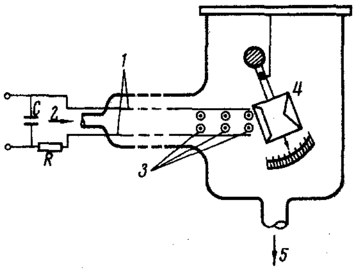

Fig. 1

$$W_v = \frac{1}{2} m(t) v^2(t), \quad W_L = \frac{1}{2c_0^2} L(t) i^2(t),$$

$$W_B = \frac{1}{c_0} iBsx(t) \qquad W_C = \frac{1}{2C}\left[\int_0^t i(\tau)d\tau\right]^2. \qquad (1.1)$$

The nonlinear equations of motion in the generalized velocities v(t) and i(t) (the equation of momenta and the generalized Ohm's law) with allowance for Joule dissipation and changes in the mass of the current layer as a result of erosion of the guide electrodes in the form

$$m(t) = m_0 + K\int_0^t |i(\tau)|\,d\tau$$

were reduced to a single nonlinear equation describing the voltage oscillations in the accelerating circuit,

$$\frac{d^2U}{dt^2} + \left[2r - \varepsilon U\left(\varepsilon_1 + \frac{dU}{dt}\right)\right]\frac{dU}{dt} + c_1^2 U = 0$$

$$r = \frac{Rc_0^2}{2L_0}, \quad c_1^2 = \frac{c_0^2}{CL_0}, \quad \varepsilon_1 = -\frac{2Bsc_0}{lC},$$

$$\varepsilon = \frac{(lC)^2}{2c_0^2 L_0 (m_0 + KCU_0)}. \qquad (1.2)$$

Here R is the total ohmic resistance of the accelerating circuit, $L_0$ is the initial inductance, C is the capacitor's capacitance, $l$ is the inductance per unit length of the guide electrodes, B is the external magnetic field, s is the distance between electrodes, $m_0$ is the initial mass of the plasma bunch, K is the electrochemical equivalent of the electrode material (the "erosion factor" [2]), and $c_0$ is the velocity of light.

Equation (1.2), which describes the intrinsic oscillatory process with a linear restoring force and nonlinear friction, does not have an exact solution [3]. Using the asymptotic method applicable to nonlinear strongly damped oscillations proposed in [1], we can obtain an approximate solution which satisfies Eq. (1.2) to within quantities of the order $\varepsilon^n$ and differs from the exact solution by an amount smaller than $t\varepsilon^n$ (n = 1, 2, 3, ...). This solution for n = 2 (the second approximation) in the absence of an external magnetic field can be written as

$$U(t) = N(e^{2rt} + \alpha)^{-1/2} \cos\left[\frac{4\omega t}{3} - \frac{\omega}{6r}\ln\frac{\alpha + e^{2rt}}{1+\alpha}\right],$$

$$N^2 = \frac{8U_0^2}{8 + 3\varepsilon U_0^2}, \qquad \alpha = -\frac{3\varepsilon U_0^2}{8 + 3\varepsilon U_0^2}, \qquad \omega^2 = c_1^2 - r^2. \quad (1.3)$$

The quantities characterizing the acceleration process in which we are interested, namely the electrical current i(t) flowing in the plasma, the velocity v(t) of the current layer, the momentum I(t) and mass m(t) of the plasma bunch, and the energy conversion factor η(t) in the accelerator, are completely defined by Eq. (1.3) and can be investigated by the familiar methods of numerical analysis,

$$i(t) = -C\frac{dU}{dt}, \qquad v(t) = \frac{lC^2}{2c_0^2 m(t)}\int_0^t \left(\frac{dU}{d\tau}\right)^2 d\tau,$$

$$I(t) = v(t)\,m(t)$$

$$m(t) = \sum_{\alpha=1}^{k} m(t_\alpha) + KC|U(t_k) - U(t)|,$$

$$\eta(t) = \frac{I^2(t)}{CU_0^2 m(t)}. \qquad (1.4)$$

In the expression for m(t) the quantity $t \in [t_k; t_{k+1}]$; moreover, the values of $t_\alpha$ coincide with the successive extrema of the voltage U(t).

For qualitative analysis of the behavior of quantities in Eq. (1.4) we can use approximate expressions in the form of rapidly converging alternating-sign series for which the approximation error can be readily ascertained. In the practically common case of moderate voltages and capacitances (the criterion is the condition $|\alpha| \ll 1$) the electrical and kinematic characteristics of the acceleration process can be represented approximately in a form convenient for analysis and computation,

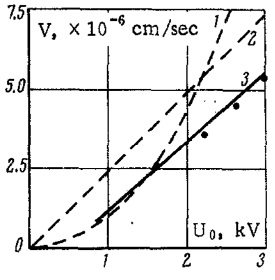

Fig. 2

$$U(t) = \frac{U_0}{\sqrt{1-\beta^2}} e^{-rt}\cos(\omega t - \varphi),$$

$$m(t) = \sum_{k=0}^{n} m(t_k) + \frac{KCU_0 \cos(\omega t_n - \varphi)\, e^{-rt_n}}{\sqrt{1-\beta^2}} \times$$

$$\times\left[1 - e^{-r(t+t_n)}\frac{\cos(\omega t + \varphi)}{\cos(\omega t - \varphi)}\right],$$

$$v(t) = \frac{lCU_0}{4c_0^2 R(1-\beta^2)\,m(t)} \times$$

$$\times\left\{(1 - e^{-2rt}) - \beta^2\left[1 + e^{-2rt}\frac{1}{\beta}\sin(2\omega t - \varphi)\right]\right\},$$

$$\beta^2 = \frac{r^2}{c_1^2} = \left(\frac{c_0}{2}\right)^2\frac{R^2C}{L_0}, \qquad \varphi = \operatorname{arc\,tg}\frac{r}{\omega}. \qquad (1.5)$$

The criterion of the effect of variations in the mass of the plasma bunch on the dependences of current layer velocity and accelerating efficiency on the electrical parameters of the accelerating circuit is the number $\chi = m_0/KCU_0$. The quantity $\chi$ represents the ratio of the mass of the current layer at the instant of current formation to the mass increment which would result if the entire initial charge on the capacitor were passed through the plasma. For $\chi \ll 1$ the effect of the mass change is large ("strong erosion"); for $\chi \gg 1$ it is small ("weak erosion").

The velocity v(t) of the current layer rapidly reaches its maximum value

$$v_{\max} \approx \frac{lU_0}{2c_0^2RK(1+\chi)} \qquad (1.6)$$

which depends essentially on the initial voltage $U_0$ (it is approximately proportional to $U_0^2$ with weak erosion, when $\chi \gg 1$, and is linear in $U_0$ with strong erosion, when $\chi \ll 1$), on the inductance increment per unit length of the guide electrodes (it is proportional to $l$), and need not depend on capacitor capacitance (for $\chi \gg 1$ the limiting velocity is linear in C; for $\chi \ll 1$, i.e., with "strong erosion", the dependence on C vanishes).

The momentum acquired by the plasma during acceleration tends asymptotically to its limiting value

$$I_{\max} \approx \frac{lCU_0^2}{4c_0^2R} \qquad (1.7)$$

which does not depend on the intensity of erosion.

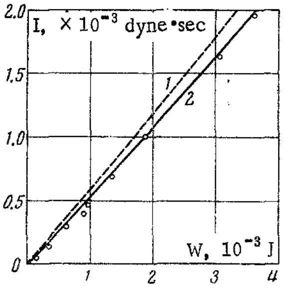

Fig. 3

The electrical energy conversion factor (the acceleration efficiency) reaches its maximum value

$$\eta_{\max} \approx \frac{l^2U_0}{(4c_0^2R)^2K(1+\chi)}\,. \qquad (1.8)$$

In the case of weak erosion ($\chi \gg 1$) the acceleration efficiency is proportional to the initial electrical energy stored in the capacitor and decreases as the mass of the plasma bunch increases. In the case of strong erosion ($\chi \ll 1$) the acceleration efficiency depends weakly on the initial energy (it is linear in $U_0$ and is completely independent of the capacitor capacitance); it does not depend on the mass of the current layer. The effect of inductance increments is large in both cases ($\eta \sim l^2$).

In the presence of a sufficiently strong external magnetic field the limiting velocity of the current layer and the acceleration efficiency can increase substantially [1]. The criterion for the magnitude of the external magnetic field is the ratio of the Lorentz force function to the energy of the intrinsic magnetic field,

$$\frac{2c_0sB}{i_{\max}\,[l + L_0/x_0]} \gtrsim 1 \qquad (1.9)$$

where $x_0$ is the length of the guide electrodes and s is the distance between the electrodes.

This effect of the magnetic field has to do with the fact that the increase in the internal energy of the plasma is limited in the external field due to reduced Joule dissipation as a result of the induced counter emf. Hence, a considerable portion of the electrical energy converted into heat in the absence of an external magnetic field is converted into kinetic energy of the accelerated plasma in the presence of such a field.

With sufficiently long guide electrodes under conditions of diffuse dispersion of the plasma bunch, the velocity v(t) of the current layer can decline markedly after reaching its maximum. This is due to the energy dissipation during interaction of particles diffusing from

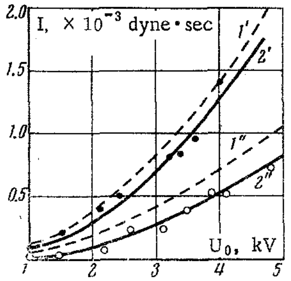

Fig. 4

the plasma bunch with the electrodes and accelerator walls (this interaction takes the form of viscous friction). The velocity is also affected by increases in the mass of the current layer which drags along particles liberated from the electrodes as a result of ionic bombardment and Joule heating (the "plasma piston"); the dragging effect is due to recharging mechanisms and to elastic collisions. The effects involved in the competing processes of buildup and diffusion of the mass of the plasma bunch and in viscous friction during acceleration, as well as the conditions under which these processes are significant, are considered in [4]. Diffusive dispersion of the plasma bunch can also lower the momentum of the accelerated plasma and the acceleration efficiency, although the quantitative dependence of these quantities on the electrical and geometric parameters of the accelerator remains unchanged.

## §2. THE EXPERIMENT

The kinematics of the plasma was investigated in a rail-type plasma accelerator (Fig. 1). The guide electrodes took the form of parallel copper rods (1) 2.5 cm wide, 0.15 cm thick, and 37 cm long. The distance between electrodes was variable from 0.5 to 5.5 cm. The plasma source was the electrical current which flowed between the guide electrodes upon discharging of the capacitor battery (capacitance C = 50–300 μF). In order to initiate current flow, the forward front of the gas fed into the space (2) between the electrodes (at different potentials) was broken down. The initial voltage across the capacitor plates ranged from 0.5 to 7 kV. The pressure of residual gases in the accelerator did not exceed $10^{-4}$ mm Hg. The velocity of the plasma was determined by means of double probes (3) mounted in the space between the guide electrodes. These probes registered the instants of arrival of the ionization front. The momenta of the plasmas were measured by means of ballistic pendulums (4) with characteristic oscillation periods of 3–4.5 sec. These enabled us to record

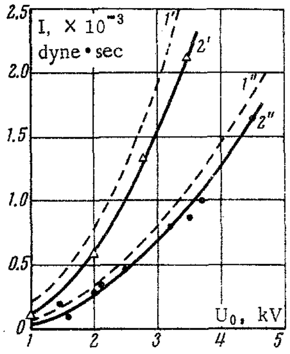

Fig. 5

The results of our experimental measurements of the velocities and momenta of plasmas and of the electrical energy conversion factor in the accelerator appear in Figs. 2–7. The same figures also contain the corresponding theoretical curves.

## §3. DISCUSSION OF THE RESULTS

Analysis of the theoretical and experimental results enabled us to draw the following conclusions.

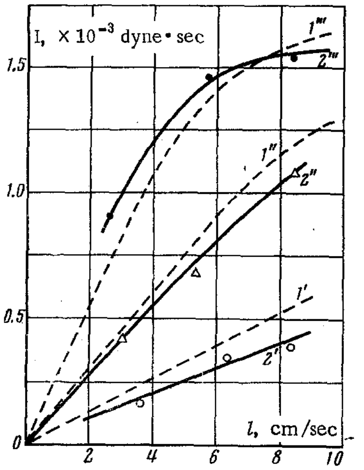

Fig. 6

1°. The limiting plasma velocity with weak erosion of the guide electrodes ($\chi \gg 1$) depends linearly on the initial electrical energy and on the inductance increment per unit length of the accelerating circuit. It is inversely proportional to the mass of gas captured by the current layer. In the case of strong erosion ($\chi \ll 1$) the limiting velocity does not depend on the mass of gas fed into the accelerator and is weakly dependent on the initial electrical energy. The dependence on the capacitance vanishes, while the linear dependence on the initial voltage remains unaltered.

This difference in the character of the dependence of the plasma velocity on the parameters of the accelerating circuit is due to the increase in mass of the plasma during its acceleration and to the consequent decrease in the acceleration of the plasma.

The dependence of the velocity of the plasma in the rail-type accelerator on the initial voltage across the accelerating circuit capacitor is shown in Fig. 2. Curve 1 was computed for the case of weak erosion, and curve 2 for strong erosion. Solid curve 3 was constructed through the experimental points. We see that for small voltages ($U_0 \leqslant 10^3$ V), when erosion can be considered weak ($\chi \ll 1$), the dependence of the plasma velocity on the initial voltage appears to be nearly quadratic (curve 1). At higher voltages the measured velocity of the plasma depends linearly on the voltage and is qualitatively similar to theoretical curve 2. This is typical of the acceleration mechanism with strong erosion of the guide electrodes. We therefore established experimentally that the velocity of the current layer was linearly dependent on the initial voltage in the voltage range studied ($U_0 \in \in (1;\ 3) \times 10^3$ V). This indicates that the erosion mechanism plays an important role in the acceleration process.

2°. The momentum acquired by the plasma during acceleration does not depend on the mass accelerated, but it is linearly dependent on capacitance and the inductance increment per unit length of the accelerating circuit ($\sim l$), and is proportional to the square of the initial voltage. This dependence of the plasma momentum on the accelerating circuit was adequately confirmed by experiment.

Figure 3 shows theoretical (1) and experimental (2) curves of the plasma momentum on the electrical energy stored in the capacitor. In the energy range studied both curves were straight, passed through the origin, and lay fairly close to each other.

Figures 4 and 5 show the momentum of the plasma as a function of the initial voltage across the accelerating circuit capacitor. The dashed curves are theoretical; the solid curves pass through the experimental points. The parameters are the capacitance C of the capacitor (Fig. 4) and the ohmic resistance of the accelerating circuit (Fig. 5). Curves 1' and 2' in Fig. 4 correspond to $C = 3 \times 10^2$ μF, and curves 1'' and 2'' to $C = 1.5 \times 10^2$ μF. Curves 1' and 2' in Fig. 5 correspond to $R = 2 \times 10^{-2}$ ohm, and curves 1'' and 2'' to $R = 4 \times 10^{-2}$ ohm. The experimental points lie near to, and somewhat below, the theoretical curves, indicating a parabolic dependence of the plasma momentum on the initial voltage. This agrees with the theoretical conclusions cited above. The shapes of the curves in Figs. 3–5 do not depend on the intensity of guide electrode erosion; the momentum of the plasma must increase rapidly with increasing energy stored in the accelerating circuit.

The inductance increment per unit length of the accelerating circuit (the quantity $l$) has a strong effect on the plasma momentum. This is evident from Fig. 6, where curves 1' and 2' were obtained for $U_0 = 2$ kV, curves 1'' and 2'' for $U_0 = 4$ kV, and curves 1''' and 2''' for $U_0 = 6$ kV. The parameter in this case is the initial voltage. As above, the dashed curves are theoretical; the experimental points are connected by the solid curve, which is qualitatively similar to the theoretical curves. Initially, both the theoretical and the experimental curves are close to straight lines passing through the origin. The experimental curves then exhibit a more or less marked (depending on the value of the parameter $U_0$) tendency towards a growth slowdown. This was due to the fact that we varied $l$ by increasing the distance between the guide electrodes. As a result of the increased length of the current layer, the total ohmic resistance of the accelerating circuit increased and lowered the momentum of the plasma. This fact was allowed for in constructing theoretical (dashed) curves 1' and 1'', which also tend to grow more slowly with sufficiently large $l$.

3°. The electrical energy conversion factor in the plasma accelerator with weak erosion ($\chi \gg 1$) depended linearly on the initial electrical energy of the accelerating device and was proportional to the square of the inductance increment ($\sim l^2$). In the case of strong erosion ($\chi \ll 1$) the energy conversion coefficient continues to increase rapidly with increasing $l$ and can be completely independent of the electrical energy, since the dependence on capacitance vanishes, leaving only the linear dependence on the initial voltage.

Figure 7 shows the energy convergence coefficient in the plasma accelerator as a function of the initial voltage across the capacitor. Dashed curve 1 represents the theoretical conversion factor for weak erosion ($\chi \gg 1$); dashed curve 2 was completed for the case of strong erosion ($\chi \ll 1$); solid straight line 3 passes through the experimental points. We see that curve 3 is qualitatively similar to curve 2. Thus, experiment confirms the linear dependence of the energy conversion factor on the initial voltage in our range of initial voltages $U_0 \in (1;\ 5) \times \times 10^3$ V. This is typical of the mechanism of acceleration along strongly erodable electrodes for which the mass of the plasma on termination of acceleration is much larger than the mass of gas admitted into the accelerator. The theoretical curves in Figs. 2–7 usually lie somewhat above the experimental curves. This is due to various dissipation processes not allowed for in our theory (e.g., diffusive dispersion of the plasma mass and viscous dissipation in the ambient medium [4]).

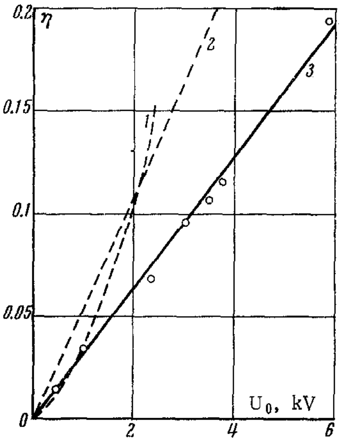

Fig. 7

End effects can play an important role in some cases. An electrical arc which sometimes continues to burn at the electrode ends after escape of the plasma can contribute to the measured characteristics of the accelerated plasma [5]. It is possible to neglect end effects in the case of a sufficiently long accelerator and with strong damping of the electrical oscillations in the accelerating circuit, or with escape of the plasma bunch in the neighborhood of zero current [6].

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